A New Class of Eigenfunction Expansion Methods for Fast Frequency-Domain Analysis of Waveguides

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***Abstract*—This paper presents a new class of algorithms for a fast calculation of dispersion characteristics in inhomogeneously loaded strictly bidirectional waveguides. The algorithms use the method of moments to convert the wave equation to a matrix eigenvalue problem using, as a basis, a set of known solutions determined for a few selected points from the $\omega-\beta$ diagram. Depending on the choice of points for the basis and the testing functions, several algorithms are derived, each leading to extremely simple expressions for the calculation of the matrix elements.**

***Index Terms*—Dispersion characteristics, method of moments, numerical analysis, waveguides.**

I. Introduction

FINDING dispersion characteristics of complex waveguides using frequency-domain techniques such as finite-element or finite-difference methods involves solving a complex boundary value problem for a large number of frequency or propagation constant points. Such an approach may be very inefficient, as it implies that the numerical complexity is identical at each point. Recognizing this fact, two techniques that alleviate this problem have been proposed. In the first one [1], [2], the problem is solved for modes at cutoff, which are then used as a basis for the field expansion. Using the method of moments, the wave equation is converted into a matrix eigenproblem. The solution outside cutoff is found by calculating the eigenvalues of a small dense matrix. In contrast to generalized telegraphist's equation approach [4] and the eigenmode transformation technique [5], both methods use expansion functions computed for inhomogeneous waveguides. As a result, a very small number of expansion terms yields acceptable results, even for a high-contrast step index permittivity profile.

Another approach [3] employs a technique called the asymptotic waveform evaluation (AWE), which uses the Taylor series or Padé approximation to represent the dispersion characteristics around a selected frequency point. The advantage of using these algorithms lies in their ability to reduce the computational effort. The sophisticated and often time-consuming methods are used only once—to construct the basis or to evaluate the expansion coefficients required by AWE. Once this has been done, the solution for the wide frequency range is quickly computed.

In this paper, we present several algorithms for fast calculation of dispersion characteristics and fields in inhomogeneously loaded waveguides. The algorithms are similar to those presented in [1] and [2] in the sense that they also employ the method of moments to convert the wave equation to a matrix eigenvalue problem using, as a basis, a set of known solutions of a loaded guide determined for a few selected points from the $\omega-\beta$ diagram. However, unlike in the previous methods, the choice is not limited to modes at cutoff, and the computation of matrix elements is also significantly simpler. We show that, by a careful choice of points for the basis and the testing functions, several new algorithms are derived, each leading to extremely simple expressions for the calculation of the matrix elements.

II. Theory

In this paper, we shall concentrate on a class of lossless and strictly bidirectional guides [6], which consists of structures uniform in the z-direction, laterally bounded by perfect electric or magnetic screens and inhomogeneously filled with an anisotropic medium whose (absolute) permittivity and permeability tensors are given by

$$\overleftrightarrow{\epsilon}=\begin{bmatrix}\underline{\underline{\epsilon}}_t & 0\\ 0 & \epsilon_{zz}\end{bmatrix}\quad \overleftrightarrow{\mu}=\begin{bmatrix}\underline{\underline{\mu}}_t & 0\\ 0 & \mu_{zz}\end{bmatrix}. \tag{1}$$

The wave propagation in such a guide may be described by the following equation [9]:

$$-\hat{z}\times\underline{\underline{\mu}}_t\cdot\nabla_t\frac{1}{\mu_{zz}}\nabla_t\cdot\hat{z}\times\vec{E}_t+\nabla_t\frac{1}{\epsilon_{zz}}\nabla_t\cdot\underline{\underline{\epsilon}}_t\vec{E}_t -\omega^2\hat{z}\times\underline{\underline{\mu}}_t\cdot\hat{z}\times\underline{\underline{\epsilon}}_t\vec{E}_t-\beta^2\vec{E}_t=0 \tag{2}$$

where $\hat{z}$ is a unit vector in the z-direction, $\vec{E}_t$ is the transverse electric field, and β, ω are the propagation constant and the angular frequency, respectively. Premultiplying the above equation with $-\hat{z}\times\underline{\underline{\mu}}_t^{-1}\hat{z}\times$, one gets

$$-\hat{z}\times\nabla_t\frac{1}{\mu_{zz}}\nabla_t\cdot\hat{z}\times\underline{\underline{\epsilon}}_t^{-1}\vec{D}_t-\hat{z}\times\underline{\underline{\mu}}_t^{-1}\hat{z}\times\nabla_t\frac{1}{\epsilon_{zz}}\nabla_t\cdot\vec{D}_t +\omega^2\vec{D}_t+\beta^2\hat{z}\times\underline{\underline{\mu}}_t^{-1}\hat{z}\times\underline{\underline{\epsilon}}_t^{-1}\vec{D}_t=0. \tag{3}$$

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Here, $\vec{D}_t$, the transverse electric flux density, is regarded as an unknown.

Both problems can compactly be written in the following operator form:

$$\mathbf{L}u + \omega^2 \mathbf{G}u - \beta^2 \mathbf{S}u = 0 \tag{4}$$

where u stands for $\vec{E}_t$ or $\vec{D}_t$, $\mathbf{L}$ represents differential part of the operators and $\mathbf{S}$, $\mathbf{G}$ are operators that either involve the media parameters (e.g., $\mathbf{S}$ for $\vec{D}_t$ or $\mathbf{G}$ for $\vec{E}_t$) or are simple unit operators.

The class of new algorithms discussed in this paper is based on the assumption that the above problem has been solved at N discrete points so that we know triads $\{\omega_i^2, \beta_i^2, u_i(\omega_i, \beta_i)\}$, $i = 1 \cdots N$, which satisfy equation

$$\mathbf{L}u_i = -\omega_i^2 \mathbf{G}u_i + \beta_i^2 \mathbf{S}u_i \tag{5}$$

within domains determined by the boundary conditions. It is further assumed that the solutions of the transposed problem, denoted by $\{\omega_k^2, \beta_k^{2*}, v_{k*}(\omega_k, \beta_k^*)\}$, $k = 1, \ldots, N$ have been found at these points. For noncomplex modes, these transposed solutions for the problems defined by (2) and (3) are given by [9], respectively, $v_{k*} = v_k = \hat{z} \times \vec{H}_{tk}$ and $v_{k*} = v_k = \hat{z} \times \vec{B}_{tk}$, where $\vec{H}_{tk}$ and $\vec{B}_{tk}$ is the magnetic field and flux density calculated from the corresponding electric quantities satisfying (5). If a mode is complex, the transposed solution is given by $v_{k*} = \hat{z} \times \vec{H}_{tk*}$ or $v_{k*} = \hat{z} \times \vec{B}_{tk*}$, with the asterisk in the subscript position indicating that the field corresponding to β_k^{2*} has to be substituted.

The set of known solutions u_i forms the basis for the method-of-moments solution of (4), while the solutions of a transposed problem v_{k*} are used as testing functions. The approximate solution of (4) for arbitrary ω and β is given by a truncated series

$$u(\omega, \beta) = \sum_{i=1}^{N} a_i u_i \tag{6}$$

with $a_i = a_i(\omega, \beta)$.

Substituting the above decomposition into (4), one gets

$$\sum_{i=1}^{N} a_i \left[\mathbf{L}u_i + \omega^2 \mathbf{G}u_i - \beta^2 \mathbf{S}u_i \right] = 0. \tag{7}$$

Using (5), we can now replace the first terms under the summation sign with $-\omega_i^2 \mathbf{G}u_i + \beta_i^2 \mathbf{S}u_i$, when the equation to be solved becomes

$$\sum_{i=1}^{N} a_i \Big[\left(\beta_i^2 - \beta^2\right) \mathbf{S}u_i + \left(\omega^2 - \omega_i^2\right) \mathbf{G}u_i \Big] = 0. \tag{8}$$

Taking the inner product of (8) with functions v_{k*} gives the following set of equations:

$$\underline{\underline{G}} \left(\omega^2 \underline{\underline{I}} - \underline{\underline{\Omega}}^2\right) \underline{a} = \underline{\underline{S}} \left(\beta^2 - \underline{\underline{Z}}^2\right) \underline{a} \tag{9}$$

where $\underline{\underline{\Omega}} = \mathrm{diag}[\omega_i^2]$, $\underline{\underline{Z}}^2 = \mathrm{diag}[\beta_i^2]$, $\underline{a} = [a_1, a_2 \ldots a_N]^T$ and the elements of matrices $\underline{\underline{G}}$ and $\underline{\underline{S}}$ are given by $G_{ki} = \langle \mathbf{G}u_i, v_{k*} \rangle$ and $S_{ki} = \langle \mathbf{S}u_i, v_{k*} \rangle$, respectively, where $\langle \cdot, \cdot \rangle$ denotes an inner product. The calculation of matrices $\underline{\underline{G}}$ and $\underline{\underline{S}}$ is extremely simple. For instance, taking $u_i = \vec{D}_{ti}$, we have $v_{k*} = \hat{z} \times \vec{B}_{tk*}$ and, hence, the elements of the matrices are given by

$$G_{ki} = \langle \mathbf{G}u_i, v_k \rangle = \hat{z} \cdot \int_{\Omega} \vec{D}_{ti} \times \vec{B}^*_{t_{k*}} \, d\Omega \tag{10}$$

$$S_{ki} = \langle \mathbf{S}u_i, v_k \rangle = \hat{z} \cdot \int_{\Omega} \vec{E}_{ti} \times \vec{H}^*_{t_{k*}} \, d\Omega \tag{11}$$

where Ω denotes the cross section of a guide. It is seen that the new algorithm requires the calculation of integrals involving only the z components of electromagnetic momenta and Poynting vectors describing stress–energy interactions between modal fields used in the basis [6].

Equation (9) can easily be transformed to give a generalized matrix eigenvalue equation with either ω^2 or β^2 treated as an eigenvalue. Treating ω (or β) as a varying parameter and solving the matrix problem for $\beta^2(\omega)$ [or $\omega^2(\beta)$] and $\underline{a}_i(\omega, \beta)$, one gets the approximate dispersion characteristics and field distribution for up to N modes of a waveguide of interest.

The procedure described above is common for a whole family of new techniques, which differ one from another by the criterion adopted for the choice of the basis and the type of equation [i.e., (2) or (3)] solved. Note that, thus far, nothing has indeed been said about the choice of the points for calculating the basis, i.e., ω_i and the corresponding β_i can be arbitrary selected from the points of dispersion diagram. For instance, one may choose these points so that all of them belong to dispersion curve of a mode, which is of our particular interest. In that case, all fields used in series (6) would correspond to the fields of the same mode calculated at different frequency points. Let us consider other choices. Suppose all expansion functions are calculated for the same β_0. (One important case is when $\beta_0 = 0$, i.e., when the eigenfunctions corresponding to the modes at cutoff are used in the expansion). This implies $\underline{\underline{Z}}^2 = \beta_0^2 \underline{\underline{I}}$ and (9) becomes

$$\underline{\underline{G}} \left(\omega^2 \underline{\underline{I}} - \underline{\underline{\Omega}}^2\right) \underline{a} = \underline{\underline{S}} \left(\beta^2 - \beta_0^2\right) \underline{a} \tag{12}$$

or

$$\left(\omega^2 \underline{\underline{I}} - \underline{\underline{\Omega}}^2\right)^{-1} \underline{\underline{G}}^{-1} \underline{\underline{S}} \underline{a} = \frac{1}{\beta^2 - \beta_0^2} \underline{a}. \tag{13}$$

We have now obtained a standard matrix eigenvalue problem with $(\beta^2 - \beta_0^2)^{-1}$ as an eigenvalue. The purpose of this transformation becomes obvious when one notes that if $u_i = \vec{D}_{ti}$, then $\mathbf{G} = \mathbf{I}$. Accordingly, since u_i and v_{k*} are orthogonal for fixed $\beta = \beta_0$ (v_{k*} in our algorithms denotes the solution to the transposed problem), then $G_{ki} = \langle u_i, v_{k*} \rangle = A_i \delta_{ki}$, where A_i is the normalizing constant and δ_{ik} is a Kronecker symbol. As a result, taking $\vec{D}_t$ as the unknown field and using fields computed for β_0 as a basis, one reduces the problem to the solution of

$$\underline{\underline{A}} \underline{a} = \frac{1}{\beta^2 - \beta_0^2} \underline{a} \tag{14}$$

where the elements of matrix $\underline{\underline{A}}$ are given by an extremely simple formula $A_{ik} = (\omega^2 - \omega_i^2)^{-1} s_{ik}$ with

$$s_{ik} = \frac{\hat{z} \cdot \int_\Omega \vec{E}_{ti} \times \vec{H}^*_{tk*}\, d\Omega}{\hat{z} \cdot \int_\Omega \vec{D}_{ti} \times \vec{B}^*_{ti*}\, d\Omega}. \tag{15}$$

The solution of (14) gives the characteristics in the form $\beta(\omega)$. Alternatively, one may transform (12) so that ω^2 is an eigenvalue and the problem to be solved is

$$\underline{\underline{B}}\underline{a} = \omega^2 \underline{a} \tag{16}$$

with $B_{ik} = (\beta^2 - \beta_0^2) s_{ik} + \omega_i^2 \delta_{ik}$.

Another pair of algorithms is obtained if $u = \vec{E}_t$ and the basis is defined by $\{u_i, \omega_0, \beta_i\}$. In other words, eigensolutions of (2) evaluated for $\omega = \omega_0$ are used to represent the electric field. If this is the case, then $\mathbf{S} = \mathbf{I}$, which entails the diagonalization of matrix $\underline{\underline{G}}$ and yields two algorithms given by the matrix eigenproblems

$$\underline{\underline{C}}\underline{a} = \frac{1}{\omega^2 - \omega_0^2}\, \underline{a} \tag{17}$$

$$\underline{\underline{D}}\underline{a} = \beta^2 \underline{a} \tag{18}$$

with the elements of matrices $\underline{\underline{C}}$ and $\underline{\underline{D}}$ given by $C_{ik} = (\beta^2 - \beta_i^2)^{-1} g_{ik}$ and $D_{ik} = (\omega^2 - \omega_i^2) g_{ik} + \beta_i^2 \delta_{ik}$ and g_{ik} defined as

$$g_{ik} = \frac{\hat{z} \cdot \int_\Omega \vec{D}_{ti} \times \vec{B}^*_{tk*}\, d\Omega}{\hat{z} \cdot \int_\Omega \vec{E}_{ti} \times \vec{H}^*_{ti*}\, d\Omega}. \tag{19}$$

Equations (9), (14), and (16)–(18) describe six algorithms that differ one from another by the choice of basis functions and the selection of an unknown and a parameter. For convenience, all algorithms are summarized in Table I. The following convention is used to designate the algorithms. The first letter denotes the type of dispersion characteristics generated by the algorithm, while one or two letters after the dash indicate the quantity (S for Poynting vectors and G for electromagnetic momenta) required to evaluate the matrix' elements. It has to be noted that yet another set of algorithms can be obtained if $\mathbf{G}^{-1} v_{k*}$ or $\mathbf{S}^{-1} v_{k*}$ are taken as testing functions. The algorithms resulting from this choice will not be discussed in this paper.

At this point, we can discuss a few important aspects of new algorithms. First of all, they all assume that the set of basis functions is complete. As shown in [7], basis functions determined from vector-wave equations (operator pencils) for $\beta_0 \neq 0$ and $\omega_0 \neq 0$ form a complete set. If the basis is formed by solving scalar TE or TM problems for $\beta_0 = 0$ or ω_0, one has to include additional terms taken form the null space of the scalar operators [6], [9]. If the basis functions are selected for arbitrary points from the dispersion diagram (for algorithms β-GS and ω-GS), the completeness cannot be inferred as easily. However, numerical tests presented in the following section and in [10] indicate that it is possible to obtain good results for a single dispersion curve using as few as four expansion terms even if all expansion points are computed for the same mode.

TABLE I
CLASSIFICATION OF ALGORITHMS

Algorithm	Unknown	Basis	Eigenvalue	Eq
β-GS	$\beta(\omega)$	$\{u_i, \omega_i, \beta_i\}$	β^2	(9)
ω-GS	$\omega(\beta)$	$\{u_i, \omega_i, \beta_i\}$	ω^2	(9)
β-S	$\beta(\omega)$	$\{u_i, \omega_i, \beta_0\}$	$(\beta^2 - \beta_0^2)^{-1}$	(14)
ω-S	$\omega(\beta)$	$\{u_i, \omega_i, \beta_0\}$	ω^2	(16)
ω-G	$\omega(\beta)$	$\{u_i, \omega_0, \beta_i\}$	$(\omega^2 - \omega_0^2)^{-1}$	(17)
β-G	$\beta(\omega)$	$\{u_i, \omega_0, \beta_i\}$	β^2	(18)

It has to be noted that the calculation of the elements of the mode-interaction matrices is very simple for all algorithms described above. The most general formula involves calculation of the coupling between the electromagnetic field expressed by momenta or Poynting vectors of basis modes. Moreover, by a special choice of the basis and testing functions, the evaluation of the matrices is further simplified by the application of orthogonality relations. In terms of the computational effort, the new algorithms can be shown to be much more economical than most of the standard ones. The approach presented in this paper has a hybrid character. An arbitrary numerical or analytical technique can be used to evaluate the basis. Since the basis is calculated at a single frequency or propagation constant point (or at most a few points, in the case of two general algorithms β-GS and ω-GS), even a time-consuming method may be used to this end. As a second step, a dense matrix eigenvalue problem is created and solved in as many points as required. If the computational effort for solving this dense problem is lower than the computation workload in a standard algorithm, the algorithms described in this paper offer a speedup of

$$S = \frac{M t_{\mathrm{std}}}{t_{\mathrm{stb}} + M t_m} \tag{20}$$

where M is a number of computation points, t_{stb} is the time required to compute the basis and set up the matrices, t_m is the time required by a dense matrix eigenvalue solver in a new algorithms at one point, and t_{std} is the time of one solution in a standard approach. When $t_{\mathrm{std}} > t_m$, (20) tends to $M t_{\mathrm{std}}/t_{\mathrm{stb}}$. It is evident that the time savings may be significant, especially when the number of points M is large. It has to be noted that similar savings in time can be expected in algorithms proposed in [1]–[3] provided M is large.

III. RESULTS

All algorithms presented in the preceding section have been tested on several waveguiding structures [9]. In this paper, we present the most representative results for the algorithms. In a first test, we shall compare the results obtained by means of a few new algorithms with reference calculations based on the finite-difference frequency-domain technique (FDFD). The dispersion curves and the corresponding modal fields of rectangular guide loaded with a dielectric slab of relative permittivity of nine were computed in the 0–20-GHz range with a grid of 40 $\times$ 20 points.

For this grid size, the FDFD method leads to a sparse matrix eigenvalue problem with the matrix size of 1540. To compute the solution with the frequency step of 50 MHz, the sparse solver

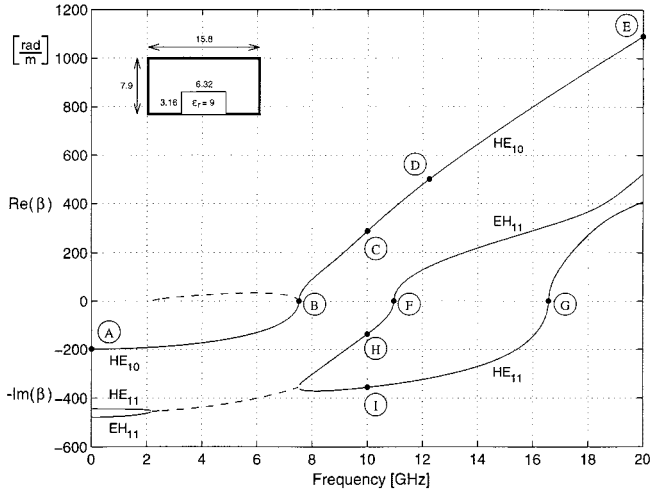

Fig. 1. Dispersion characteristics for three modes with magnetic-wall symmetry in a rectangular image guide calculated using the FDFD method. Letters A–I indicate various points at which modal fields have been calculated and used as a basis in algorithms described in this paper. The structure is shown in the inset—dimensions are in millimeters

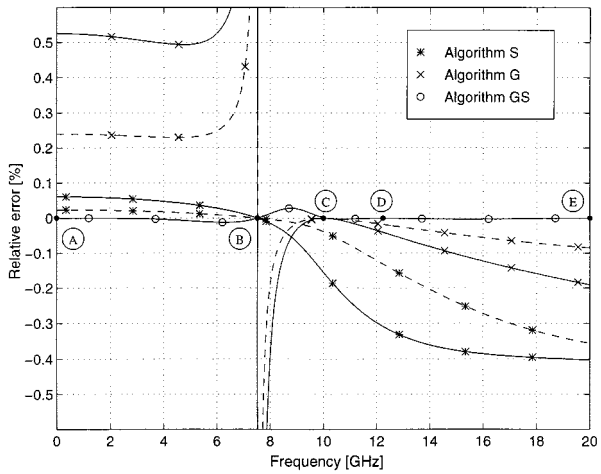

Fig. 2. Error in propagation constant for a dominant mode (relative to FDFD computations shown in Fig. 1) in three fast β algorithms. The results for algorithms β-G and β-S are plotted for the $N = 10$ (solid line) and $N = 20$ (dashed line) modes with the magnetic-wall symmetry used in series (6).

had to be applied $M = 401$ times. Thus, the characteristics obtained are shown in Fig. 1. (The modes are labeled according to the scheme used in [6].) Subsequently, three new algorithms β-GS, β-S, β-G (cf. Table I) were used to evaluate the dispersion characteristic of the dominant mode. Each of the algorithms uses a different set of eigenfunctions. Several points belonging to three sets, denoted by letters A–I, are shown in Fig. 1. In particular, for algorithm β-GS, which allows eigenfunctions for an arbitrary set of pairs ω_i, β_i, four points corresponding to the same mode, namely A, B, D, and E, were selected for the basis. Algorithm β-S was implemented for the basis calculated at $\beta_0 = 0$. The first three such points are denoted by B, F, and G in Fig. 1. Finally, for algorithm β-G, basis functions were calculated at $f = 10$ GHz. Again, the first three such points are denoted by C, H, and I in Fig. 1. Once the bases have been evaluated with the FDFD solver, the new algorithms were used to compute the dispersion characteristic at all 401 points used in

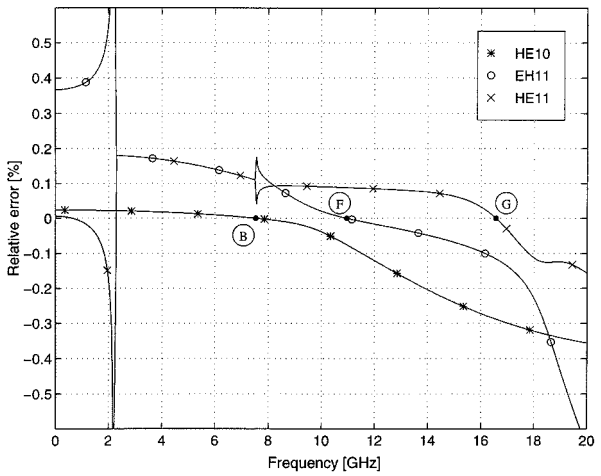

Fig. 3. Error in propagation constant for three modes (relative to FDFD computations shown in Fig. 1) in algorithm β-S using the basis constructed form $N = 20$ modes with the magnetic wall evaluated at $\beta_0 = 0$.

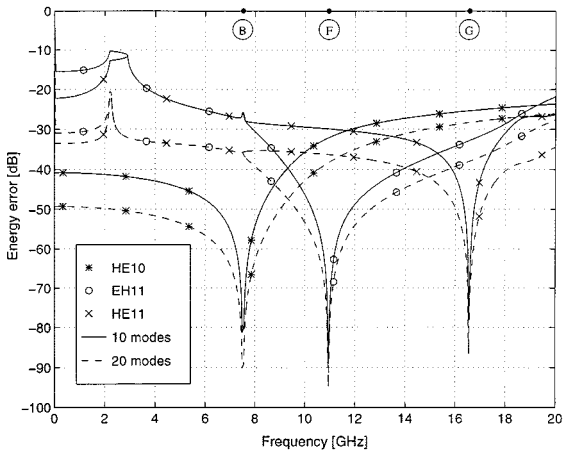

Fig. 4. Magnetic field energy error for three modes (relative to FDFD computations shown in Fig. 1) in algorithm β-S using the basis constructed form $n = 10$ (solid line) and $N = 20$ (dashed line) modes with the magnetic wall evaluated at $\beta_0 = 0$.

the reference solution. The speedup for β-S and β-G algorithms was $S \approx 401$ and $S \approx 100$ for β-GS algorithm.

The relative error in propagation constant of the dominant mode for the three algorithms is shown in Fig. 2. Algorithm β-GS used only four points in the basis. The results for algorithms β-S are β-G, shown for the bases consisting of ten (solid line) or 20 (dashed line) eigenfunctions. It is seen that all algorithms are capable of reproducing the dispersion characteristic with a very good accuracy. Since all points (A, B, D, E) used in the basis correspond to the dominant mode algorithm, β-GS gives the best results. When higher order modes are of interest, algorithm β-S should be used. Fig. 3 shows the relative error in propagation constant, obtained with this algorithm and basis size $N = 20$, for all modes shown in Fig. 1. Note that, two modes become degenerate below cutoff and produce a pair of complex waves, which exist over a finite frequency range. Algorithm β-S predicts the propagation constant for this wave with the accuracy better than 0.2%. (Only the real part is shown, but the results for the imaginary part are similar). Fig. 4 shows the

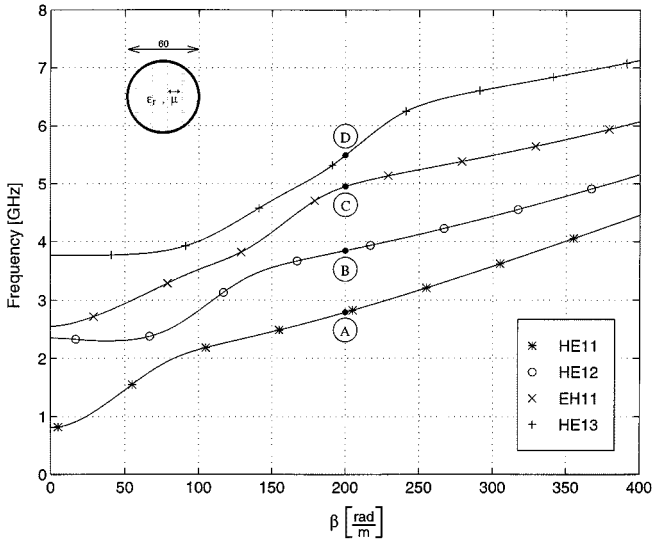

Fig. 5. Dispersion characteristics of modes in a circular waveguide homogeneously loaded with an anisotropic magnetic medium. The structure is shown in the inset—dimensions are in millimeters. The letters A–D denote the first four points of the basis in ω-S algorithm.

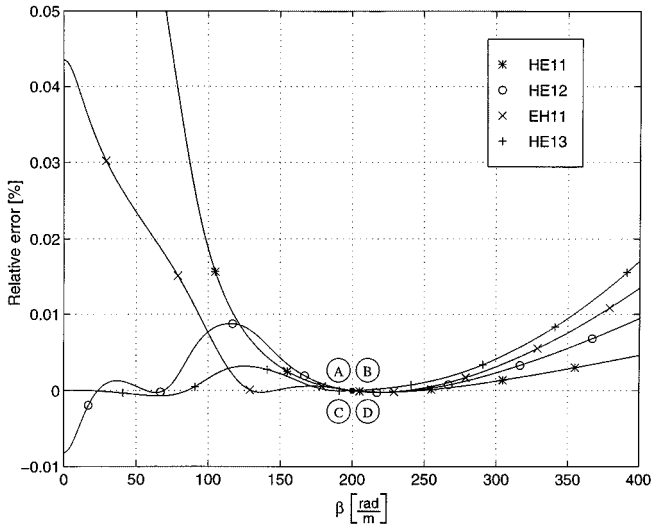

Fig. 6. Error in propagation constant for modes (relative to the solution shown in Fig. 5) in algorithm ω-S using the basis constructed form $N = 20$ modes with $m = 1$ evaluated at $\beta_0 = 200$ rad/m.

magnetic-field energy error for three modes evaluated with algorithm β-S with $N = 10$ (solid line) and $N = 20$ (dashed line). It is seen that fields of all modes are satisfactorily reproduced. The results for the electric field were even slightly better and, therefore, these plots are not presented here.

In order to illustrate the ability of the new algorithms to deal with anisotropic media, we also analyzed a circular waveguide homogeneously loaded with a magnetic medium described by the dielectric constant $\epsilon_r = 13$ and permeability tensor given by

$$\overleftrightarrow{\mu} = \mu_0 \begin{bmatrix} 1 & j0.75 & 0 \\ -j0.75 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{21}$$

Due to gyrotropic properties of the medium, the propagation constants of hybrid modes EH_{nm} and HE_{nm} depend on the sign of the index m denoting the angular variation. In this paper, we present the results for modes having the angular dependence $m = 1$. These modes were first found by solving a nonlinear dispersion equation [8] and the results, regarded as the reference solutions, are shown in Fig. 5. Subsequently, algorithm ω-S was applied with the basis consisting of $N = 20$ eigenfunctions evaluated at $\beta = 200$ rad/m. As algorithm ω-S calculates characteristics ω versus β, Fig. 6 shows the relative error in computation of the frequency corresponding to a given β. Four modes are shown. It is seen that, except for two modes near cutoff, the error is at the level of 0.01%. The largest value of the error for the HE_{11} mode occurred at $\beta = 0$ and was 0.25%. It has to be mentioned that we also calculated, in a separate test, the errors for the modes with $m = -1$. In this case, the results were even better, as the relative errors for all four modes were lower than 0.05% for the entire region $0 \leq \beta \leq 400$ rad/m.

IV. Conclusions

In this paper, we have introduced a new class of algorithms, which are intended for fast wide-band frequency-domain analysis of waveguides. The new algorithms were based on the method-of-moment solution of the wave equation. The basis and testing functions used in the algorithms satisfy the wave equation (or its transpose) for a few selected points. The careful choice of these points allows one to significantly simplify the calculation of inner products. Numerical tests showed that the new algorithms provide an accurate solution, even for the small number of functions used in the field expansion.

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